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Lecture Notes for Courses on Special Relativity at the University of Berlin and the University of Zurich, Winter Semester 1918–1919

(pp. 86–100)

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12. Lecture Notes for Courses on Special Relativity at the University of Berlin and the University of Zurich, Winter Semester 1918–1919

[11 October 1918–second half of February1919][1]

11. X[2]

[p. 1] Coulomb $\frac{ee'}{4\pi r^2}$ = Kraft. (Heav[iside] Einheiten)[3]

Bedeutet in Verallgemeinerung

$\mathrm{div}\,\mathfrak{e} = 0$ in freien Raum

$\mathrm{div}\,\mathfrak{e} = \rho$	wo el. Mengen	auch	$\mathrm{div}\,\mathfrak{h} = 0$ (bzw σ)
$\mathrm{rot}\,\mathfrak{e} = 0$	geschlossen aus Energieprinzip.		$\mathrm{rot}\,\mathfrak{h} = 0$

Erfahrungsthats. magnet. Wirkung des stationären Stromes $\mathfrak{h}\,d\mathfrak{s} = \int \mathfrak{i}_n d.$

$\mathrm{rot}\,\mathfrak{h} = \frac{1}{c}\mathfrak{i}$

Elektrost. Energie:

$$E = \int \varphi\delta\rho\, d\tau = \int \delta\left(\frac{\mathfrak{e}}{2}\right)^2 d\tau = \delta\left\{\int \frac{\mathfrak{e}^2}{2} d\tau\right.$$

Energiedichte $\eta = \frac{\mathfrak{e}^2}{2} + \frac{\mathfrak{h}^2}{2}$ (Hypothese über Verteilung der Energie.

Maxw. Hypo: Ursache des ind. Stromes ist *elektr Feld*

Erh. d. Energie auf langsam veränderlichen Strom angewendet

$$-\frac{d}{dt}\left\{\int \frac{\mathfrak{h}^2}{2} dV\right\} = \int (\mathfrak{e}\langle\ \mathfrak{h}\rangle\, \mathfrak{i}\, dV)$$

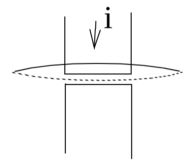
Durch partielle Integration des zweiten Gliedes:

$$\int \mathfrak{h}\left(\frac{1}{c}\frac{\partial \mathfrak{h}}{\partial t} + \mathrm{rot}\,\mathfrak{e}\right) dV = 0$$

⟨ Einfachster⟩ $\mathrm{rot}\,\mathfrak{e} = 0$ kann nicht für nichtstationäre Ströme aufrecht erhalten werden. Dafür tritt nun naturgemäss

$\mathrm{rot}\,\mathfrak{e} = -\frac{1}{c}\frac{\partial\mathfrak{h}}{\partial t}.$

Im Falle nichtstationärer sp. offener Ströme kann die Gl. $\mathrm{rot}\,\mathfrak{h} = \frac{1}{c}\mathfrak{i}$ nicht aufrecht erhalten werden. Denn man erhält durch Div. Bildung Widerspruch. $(\mathrm{div}\,\mathfrak{i} = 0)$ Erg. nun hypoth. Gl. $\frac{\mathfrak{A}}{c}$.



$$\mathrm{div}(\mathfrak{i} + A) = 0$$

$$\mathrm{div}\,\mathfrak{i} + \frac{\partial\rho}{\partial t} = 0 \text{ oder } \mathrm{div}\left(\mathfrak{i} + \frac{\partial\mathfrak{e}}{\partial t}\right) \text{ also } \mathrm{rot}\,\mathfrak{h} = \frac{1}{c}\left(\mathfrak{i} + \frac{\partial\mathfrak{e}}{\partial t}\right)$$

Es ergeben sich also die Gl. [p. 2]

$$\mathrm{rot}\,\mathfrak{h} = \frac{1}{c}\left(\frac{\partial\mathfrak{e}}{\partial t} + \mathfrak{i}\right) \qquad \mathrm{rot}\,\mathfrak{e} = -\frac{1}{c}\frac{\partial\mathfrak{h}}{\partial t}$$

$$\mathrm{div}\,\mathfrak{e} = \rho \qquad \mathrm{div}\,\mathfrak{h} = 0$$

Kraftdichte auf el. Ströme. Nach Impulssatz

$$\int (\mathfrak{h}\sigma + \mathfrak{k})dV = 0$$

$$\underset{\mathrm{div}\,\mathfrak{h}}{\sigma}$$

$$\mathfrak{h}_x\left(\frac{\partial\mathfrak{h}_x}{\partial x} + \frac{\partial\mathfrak{h}_y}{\partial y} + \frac{\partial\mathfrak{h}_z}{\partial z}\right)$$

part. Integration nach erst. Maxw. Gl.

$$\int \left(\mathfrak{k} - \frac{1}{c}[\mathfrak{i}, \mathfrak{h}]\right)dV = 0$$

$$\mathfrak{k}_{[m]} = \frac{1}{c}[\mathfrak{i}, \mathfrak{h}] = \rho\left[\frac{\mathfrak{q}}{c}, \mathfrak{h}\right]$$

Gesamtkraft also

$$\mathfrak{k} = \rho\left\{\mathfrak{e} + \left[\frac{\mathfrak{q}}{c}, \mathfrak{h}\right]\right\}$$

---

18. X.

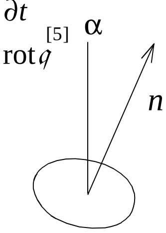
Lorentz'sche Theorie ruhender Körper[4]

$$\rho = \rho_l + \rho_d \qquad \mathfrak{i} = \mathfrak{i}_l + \mathfrak{i}_d + \mathfrak{i}_m$$

$$\rho_l = \rho \qquad \mathfrak{i}_l = \mathfrak{i}$$

$$\rho_d = \frac{\partial \mathfrak{p}}{\partial t} - \operatorname{div}\mathfrak{p} \qquad \mathfrak{i}_d = \frac{\partial \mathfrak{p}}{\partial t}$$

$$\rho_m = 0 \qquad \mathfrak{i}_m = \operatorname{rot}\mathfrak{q}$$ [5]



Ableitung für $\mathfrak{i}_m$:

Magnetel. von best $\mathfrak{J}$, f u Normale $\mathfrak{n}$[6]

$$(\mathfrak{q}) = \frac{\mathfrak{J}}{c} f (N)\mathfrak{n} \qquad \mathfrak{q}_d = \frac{\mathfrak{J}}{c} f N \cos n\alpha$$

Strom, welches $d\mathfrak{s}$ umgibt[7]

$$\mathfrak{J} \cdot (f \cos(n d\mathfrak{s}) |d\mathfrak{s}|) \cdot (N) = (\mathfrak{q}) d.$$

Für alle Magnet[--------]. $\sum (\mathfrak{q}) d\mathfrak{s}$ ~~$\mathfrak{q} d\mathfrak{s}$~~

Strom durch Fläche $\mathfrak{q} d\mathfrak{s} = c \int \operatorname{rot}_n \mathfrak{q}\, d.$[8]

Daraus $\mathfrak{i}_m = \operatorname{rot}\mathfrak{q}$

Eingesetzt

$$\operatorname{rot}(\mathfrak{h} - \overset{\mathfrak{m}}{\cancel{\mathfrak{q}}}) = \frac{1}{c}\frac{\partial(\mathfrak{e} + \mathfrak{p})}{\partial t} + \frac{1}{c}\mathfrak{i} \qquad \operatorname{rot}\mathfrak{e} = -\frac{1}{c}\frac{\partial \mathfrak{h}}{\partial t} \qquad \mathfrak{i} = \sigma\mathfrak{e}$$

$$\mathfrak{p} = (\varepsilon - 1)\mathfrak{e}$$

$$\operatorname{div}(\mathfrak{e} + \mathfrak{p}) = \rho \qquad \operatorname{div}\mathfrak{h} = 0 \qquad \mathfrak{m} = (\mu - 1)(\mathfrak{h} - \mathfrak{m})$$

$$\mathfrak{H}.$$

[p. 3] 27. X. & ⟨ 2. X.⟩

Versuche von Roland,[9] Röntgen und Eichenwald,[10] Wilson,[11] Fizeau'scher Versuch[12]; Theorie nachher. Hierzu Strom $\mathfrak{i}_l$ und $\mathfrak{i}_d$ nötig.

2. XI.

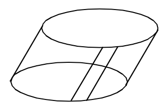
$$\mathfrak{i} = \mathfrak{i}_l + \mathfrak{i}_d \qquad \rho = \rho_e + \rho_d$$

$$\mathfrak{i}_l = \mathfrak{i} + \mathfrak{q} \langle \operatorname{div}\mathfrak{p} \rangle \rho \qquad \rho \quad -\operatorname{div}\mathfrak{p}$$

Berechnung von $\mathfrak{i}_d$

1) $\frac{\partial \mathfrak{p}}{\partial t}$

2) Beitrag bei zeitlich konstantem $\mathfrak{p}$ wegen $\mathfrak{q}$[13]



$$-\int \operatorname{div}\mathfrak{p}\ \mathfrak{q}_n'\ dS + \left(\int_{S'} - \int_{S}\right)(\mathfrak{p}_n dS \qquad = -dt\int \operatorname{rot}_n[\mathfrak{q},\ \mathfrak{p}]dS$$

Nun ist aber ⟨ nach⟩

$$\operatorname{div}\mathfrak{p}\ \mathfrak{q}_n'\ dS) = \int_{S'} - \int_{S} \circ \quad + \int \mathfrak{p}_n[\langle\ \mathfrak{q}\rangle d\mathfrak{s},\ \langle$$

$$\int d\mathfrak{s}[\mathfrak{q}_n',\mathfrak{p}]$$

$$\int \operatorname{rot}_n[\mathfrak{q}_n',\mathfrak{p}]dS$$

also $\rho_{\text{tot}} = \rho - \operatorname{div}\mathfrak{p}$

$$\mathfrak{i}_{\text{tot}} = \mathfrak{i} + \mathfrak{q}\rho - \operatorname{rot}[\mathfrak{q},\ \mathfrak{p}]$$

Lorentz'sche Gleichungen lauten also

$$\operatorname{rot}\left\{\mathfrak{h} + \left[\frac{\mathfrak{q}}{c},\ \mathfrak{p}\right]\right\} = \frac{1}{c}\frac{\partial \mathfrak{e} + \mathfrak{p}}{\partial t} + \frac{1}{c}\mathfrak{i} + \frac{\mathfrak{q}}{c}\rho \qquad \operatorname{rot}\mathfrak{e} = -\frac{1}{c}\frac{\partial \mathfrak{h}}{\partial t}$$

$$\operatorname{div}(\mathfrak{e} + \mathfrak{p}) = \rho \qquad \operatorname{div}\mathfrak{h} = 0.$$

$$\mathfrak{i} = \sigma\left(\mathfrak{e} + \left[\frac{\mathfrak{q}}{c},\ \mathfrak{h}\right]\right)$$

$$\mathfrak{p} = (\varepsilon - 1)\left(\mathfrak{e} + \left[\frac{\mathfrak{q}}{c},\ \mathfrak{h}\right]\right)$$

Bei Fizeau'schen Versuch keine Leitung[14]

[p. 4]

$$\operatorname{rot}\left\{\mathfrak{h} + \left[\frac{\mathfrak{q}}{c},\ \mathfrak{p}\right]\right\} = \frac{1}{c}\left(\frac{\partial \mathfrak{e}}{\partial t} + \frac{\partial \mathfrak{p}}{\partial t}\right) \qquad \operatorname{rot}\mathfrak{e} = -\frac{1}{c}\frac{\partial \mathfrak{h}}{\partial t}$$

nur $\mathfrak{e}_y$, $\mathfrak{p}_y$, $\mathfrak{h}_z$ von null verschieden Funkt von $(x - Vt)$

Mat Bed nach x diff

$$+\mathfrak{b}' + \frac{q}{c}\mathfrak{p}' = +\frac{V}{c}\mathfrak{b}'(\mathfrak{e}' + \mathfrak{p}') \qquad\Big|\qquad \mathfrak{p}' = (\varepsilon - 1)\left(\mathfrak{e}' - \frac{q}{c}\mathfrak{b}'\right)$$

$$\mathfrak{e}' = \frac{V}{c}\mathfrak{b}'$$ [15]

$$+\frac{V}{c}$$

$$\frac{V}{c}\mathfrak{e}' - \frac{q}{c}\mathfrak{p}' - \mathfrak{b}' = 0$$

$$\mathfrak{e}' \qquad -\frac{V}{c}\mathfrak{b}' = 0$$

$$\mathfrak{e}' - \frac{1}{\varepsilon - 1}\mathfrak{p}' - \frac{q}{c}\mathfrak{b}' = 0$$

[16]

$$\begin{vmatrix} \frac{V}{c} & \frac{q}{c} + \frac{V}{c} & 1 \\ 1 & 0 & \frac{V}{c} \\ 1 & \frac{1}{\varepsilon - 1} & \frac{q}{c} \end{vmatrix} = 0$$

$$+\frac{V^2}{c^2}$$

$$2\frac{Vq}{c^2} + \frac{1}{\varepsilon - 1} - \frac{V^2}{c^2(\varepsilon - 1)} = 0 \quad = 0$$

[17]

$$2\frac{Vq}{c^2}(\varepsilon - 1) + 1 - \varepsilon\frac{V^2}{c^2} = 0$$

$$V^2 = \frac{c^2}{\varepsilon}$$

$$V = V_0 + \Delta \qquad \varepsilon\frac{(V_0 + \Delta)^2}{V_0^2} = \varepsilon\left(1 + 2\frac{\Delta}{V_0}\right)$$

$$\frac{c}{\sqrt{\varepsilon}}$$

[18]

$$2\frac{V_0 q}{c^2}(\varepsilon - 1) = 2\frac{\Delta}{V_0}$$

$$\Delta = n^2\frac{(\varepsilon - 1)}{n^2}q = \left(1 - \frac{1}{n^2}\right)q \qquad\Big|\qquad V = V_0 + \left(1 - \frac{1}{n^2}\right)q.$$

9. XI fiel aus wegen Revolution.[19]

16. XI.
23. XI. } Lorentz-Transformation

30. XI Starre Körper und Uhren

7. XII Additionstheorem
der Geschwindigkeit
Minkowski's Interpretation
der Lorentz-Transformation.

14 XII Relativitätsprinzip und
Lorentz-Transformation
Vektoren & Tensoren als Hilfsmittel der Theorie.
Theorie der Tensoren. [p. 5]

21. XII

Symmetrische & antisymmetrische Tensoren im dreidimensionalen Raum. Spezielle T. $\delta_{\mu\nu}$ u $\delta_{\rho\sigma\tau}$ Differential-Operationen.

4. I. 19.[20]

Minkowski. Ponderable Körper

Zunächst im Lorentz'schen Sinne ergänzt.
Lorentz für ruhende Körper

$\mathrm{rot}(\mathfrak{h} - \mathfrak{m}) = \frac{\partial \mathfrak{e} + \mathfrak{p}}{\partial t} + \mathfrak{i}$	$\mathrm{rot}\,\mathfrak{e} + \frac{\partial \mathfrak{h}}{\partial t} = 0$
$\mathrm{div}(\mathfrak{e} + \mathfrak{p}) = \rho$	$\mathrm{div}\,\mathfrak{h} = 0$

Für Ruhe $-\mathfrak{m}_x \quad -\mathfrak{m}_y \quad -\mathfrak{m}_z \quad -i\mathfrak{p}_x \quad -i\mathfrak{p}_y \quad -i\mathfrak{p}_z$
$\qquad p_{23} \quad p_{31} \quad p_{12} \quad p_{14} \quad p_{24} \quad p_{34}$

$$\frac{\partial(f_{\mu\nu} + p_{\mu\nu})}{\partial x_\mu} = \mathfrak{I}_\mu$$ [21] (Vierervekt. des Leitungsstromes)

$$\frac{\partial f_{\mu\nu}}{\partial x_\rho} + \frac{\partial f_{\nu\rho}}{\partial x_\mu} + \frac{\partial f_{\rho\mu}}{\partial x_\mu} = 0$$ [22]

Materie-Bedingungen

$p_{\mu\nu}\cancel{\mathfrak{I}_\mu}u_\nu$	$p_{12}\cancel{\mathfrak{I}_2}u_2 + p_{13}\cancel{\mathfrak{I}_3}u_3 + p_{14}\cancel{\mathfrak{I}_4}u_4$	$-\mathfrak{m}_z\frac{\mathfrak{p}_y}{\sqrt{\ }} + \mathfrak{m}_y\frac{\mathfrak{p}_z}{\sqrt{\ }}\langle\ +\rangle - i\,\mathfrak{p}_x\frac{i\langle\ c\rangle}{\sqrt{\ }}$ [23]
$f_{\mu\nu}\cancel{\mathfrak{I}_\nu}u_\nu$	$f_{12}u_2 + f_{13}u_3 + f_{14}u_4$	—— —— [24] $-i\,\mathfrak{e}_x\frac{i}{\sqrt{\ }}$

$$\boxed{(\varepsilon - 1)f_{\mu\nu}u_\nu = p_{\mu\nu}u_\nu}$$

$$\mathfrak{b} \qquad \mathfrak{m}$$

$$\mathfrak{m} = \mu(\mathfrak{b} - \mathfrak{m})$$

$$\mathfrak{m}(1+\mu) = \mu\mathfrak{b}$$

$$\mathfrak{m} = \frac{\mu}{1+\mu}\mathfrak{b}$$ [26]

$$-\frac{\mu}{1+\mu}\cdot\frac{1}{[\,|\,|\,]}\left|\begin{matrix} p_{23}\mathfrak{J}_4 + p_{34}\mathfrak{J}_2 + p_{42}\mathfrak{J}_3 & \left|-\mathfrak{m}_x\cdot\frac{\mathfrak{i}}{\sqrt{\ }}\right) \\ f_{23}\mathfrak{J}_4 + \cdot + \cdot & \left|\ \mathfrak{h}_1\frac{i}{\sqrt{\ }}\right. \end{matrix}\right.$$ [25]

$$p_{\mu\nu}\mathfrak{J}_\mu u_\rho + \cdot + \cdot = -\frac{\mu}{1+\mu}(f_{\mu\nu}u_\rho + \cdot + \cdot)$$ [27]

$$\left(-1+\frac{1}{1+\mu}\right)$$

$$(p_{\mu\nu}+f_{\mu\nu})u_\rho + \cdot + \cdot = -\frac{1}{1+\mu}(f_{\mu\nu}u_\rho + \cdot + \cdot)$$

[p. 6] Ende des Kollegs am Heftende (in Zürich notiert).[28]

[p. 7]

$$i \qquad\qquad f_{23}$$

$$\frac{\partial\mathfrak{h}_z}{\partial y} - \frac{\partial\mathfrak{h}_y}{\partial z} - \frac{1}{c}\frac{\partial\mathfrak{e}_x}{\partial t} = \frac{1}{c}\mathfrak{i}_x \qquad \frac{\partial\mathfrak{e}_z}{\partial y} - \frac{\partial\mathfrak{e}_y}{\partial z} + \frac{1}{c}\frac{\partial\mathfrak{h}_x}{\partial t} = 0 \qquad \frac{\partial f_{23}}{\partial x_4} - \frac{\partial f_{34}}{\partial x_2} + \frac{1}{c}\frac{\partial f_{41}}{\partial x_2} = 0$$

$$\frac{\partial\mathfrak{h}_x}{\partial z} - \frac{\partial\mathfrak{h}_z}{\partial x} - \frac{1}{c}\frac{\partial\mathfrak{e}_y}{\partial t} = \frac{1}{c}\mathfrak{i}_y \qquad \frac{\partial\mathfrak{e}_x}{\partial z} - \frac{\partial\mathfrak{e}_z}{\partial x} + \frac{1}{c}\frac{\partial\mathfrak{h}_y}{\partial t} = 0 \qquad \text{———————}$$

$$\frac{\partial\mathfrak{h}_y}{\partial x} - \frac{\partial\mathfrak{h}_x}{\partial y} - \frac{1}{c}\frac{\partial\mathfrak{e}_z}{\partial t} = \frac{1}{c}\mathfrak{i}_z \qquad \text{— — — — —} \qquad \text{———————}$$

$$\frac{\partial\mathfrak{e}_x}{\partial x} + \frac{\partial\mathfrak{e}_y}{\partial y} + \frac{\partial\mathfrak{e}_z}{\partial z} = \rho \qquad \frac{\partial\mathfrak{h}_x}{\partial x} + \frac{\partial\mathfrak{h}_y}{\partial y} + \frac{\partial\mathfrak{h}_z}{\partial z} = 0 \qquad \text{———————}$$

Erstes System anders angeordnet

$$\cdot \quad + \frac{\partial\mathfrak{h}_z}{\partial y} + \frac{\partial-\mathfrak{h}_y}{\partial z} + \frac{1}{c}\frac{\partial-\mathfrak{e}_x}{\partial t} = \frac{1}{c}\mathfrak{i}_x$$

$$-\frac{\partial\mathfrak{h}_z}{\partial x} + \cdot \quad + \frac{\partial\mathfrak{h}_x}{\partial z} + \frac{1}{c}\frac{\partial-\mathfrak{e}_y}{\partial t} = \frac{1}{c}\mathfrak{i}_y$$

$$\text{— — — — — — —}$$

$$\frac{\partial(-i\mathfrak{e}_x)}{\partial x} + \frac{\partial-i\mathfrak{e}_y}{\partial y} + \cdot \qquad = i\rho$$

f_{23}	f_{31}	f_{12}	f_{14}	f_{24}	f_{34}
$\mathfrak{h}_z$	$\mathfrak{h}_x$	$\mathfrak{h}_y$ [29]	$-i\,\mathfrak{e}_x$	$-i\,\mathfrak{e}_y$	$-i\,\mathfrak{e}_z$

$$\boxed{\frac{\partial f_{\mu\nu}}{\partial x_\nu} = \frac{1}{c}\mathfrak{J}_\nu}$$ [30]

$\mathfrak{J}_1$	$\mathfrak{J}_2$	$\mathfrak{J}_3$	$\mathfrak{J}_4$
$\frac{\mathfrak{i}_x}{c}$	$\frac{\mathfrak{i}_y}{c}$	$\frac{\mathfrak{i}_z}{c}$	$i\rho$

Analyse der Stromdichte $\rho_0 \frac{dx_\nu}{ds}$ $\boxed{ds = \sqrt{-\sum dx_\nu^2} = dt\sqrt{c^2 - q^2}}$

Fundamental-Invariante.

$$= c\,dt\sqrt{1 - \frac{q^2}{c^2}} = c\,dt\,\nu$$

Setzt man $\frac{\rho_0}{\sqrt{V}} = \rho$, so sind die vier Komp.

$\mathfrak{J}_1$	$\mathfrak{J}_2$	$\mathfrak{J}_3$	$\mathfrak{J}_4$
$\frac{1}{c}\left(\rho_0 \frac{dx_1}{dt}\right)$ ($\mathfrak{i}_x$)	$\cdot$	$\cdot$	$i\rho_0$

Erhaltungssatz der Elektrizität aus $\sum \frac{\partial \mathfrak{J}_\nu}{\partial x_\nu} = 0$.

⟨In⟩ Elektrische Gesamtmenge ist Invariante

$$\rho_0 \, \cancel{dx\,dy\,dz\,dt}\, dV_0 \, \cancel{dys}$$

Kraftdichte $\rho\,\mathfrak{e} + \left[\frac{\mathfrak{i}}{c}, \mathfrak{h}\right] = \mathfrak{k}_x$ [p. 8]

X Komponente $\rho\,\mathfrak{e}_x + \frac{\mathfrak{i}_y}{c}\mathfrak{h}_z - \frac{\mathfrak{i}_z}{c}\mathfrak{h}_y$

$$= f_{12}\mathfrak{J}_2 + f_{13}\mathfrak{J}_3 + f_{14}\mathfrak{J}_4 = \cancel{K}k_1$$

Ist erste Komponente eines Vierervektors. Wir bilden vierte Komponente

$$\mathfrak{J}_1 + \cdot + \cdot = i\mathfrak{e}_x \cdot \frac{\mathfrak{i}_x}{c} + \cdot + \cdot =$$ [31]

Man hat

$$\begin{matrix} k_1 & k_2 & k_3 & k_4 \\ \mathfrak{k}_x & \mathfrak{k}_y & \mathfrak{k}_z & \frac{il}{c} \end{matrix}$$

Wir gehen über zu Gesamtimpuls und Gesamtenergie

$$dx_1..dx_4 = \text{Invariante} = \int dVdx_4$$

$$\left.\begin{matrix} \int (\int k_1 dV) dx_4 \\ \rule{3em}{0.4pt} \\ \rule{3em}{0.4pt} \\ \int (\int k_4 dV) dx_4 \end{matrix}\right\} \text{ebenfalls Vierervektor}$$

Impulssatz $k_1 dV = \int \mathfrak{k}\, dV = \frac{dI}{d}$ [32]

Energiesatz $k_4 dV = \frac{i}{c}\int l\ dV = \frac{i}{c}\frac{dl}{d}$

Durch Integration zwischen zwei Zeitgrenzen

$$\Delta I_1 \quad \Delta I_2 \quad \cdot \quad \frac{i}{c}\Delta E$$

Vierervektor. Was für die Zunahme gilt, wird auch für die Grössen selbst gelten (Hypothese)

$$I_1 \ , \ , \ \frac{i}{c}E \qquad \text{Vierervektor.}$$

Andere Ableitung. Wird abhängen von Masse M

und Geschwindigkeit $\frac{dx_\nu}{ds}$ $\left(ds = \frac{1}{c}\sqrt{-dx^2{}_\nu} = dt\sqrt{1 - \frac{q^2}{c^2}}\right.$

$$M\frac{dx_1}{ds} \quad . . . \quad M\frac{dx_4}{ds}$$

Durch Gleichsetzung ergibt sich

$$I_1 = M\frac{q_x}{\surd}$$

- - - - - - - - -

$$E = \frac{M}{c^2} \cdot \frac{1}{\sqrt{1 - \frac{q^2}{c^2}}}$$

In der That für ruhendes System $I_1 = 0$ [p. 9]

$$\boxed{E_0 = \frac{M}{c^2}}$$

Nachweis des Impulssatzes für elektromagnetisches Feld.

$$-k_\nu = f_{\mu\nu}\mathfrak{J}_\mu$$

$$= f_{\mu\nu}\frac{\partial f_{\mu\sigma}}{\partial x_\sigma} \quad \text{(erstes Maxwell'sches System)}$$

$$= \frac{\partial}{\partial x_\sigma}(f_{\mu\nu}f_{\mu\sigma}) - f_{\mu\sigma}\frac{\partial f_{\mu\nu}}{\partial x_\sigma}$$

$$= \quad \cdot \quad + f_{\mu\sigma}\left(\frac{\partial f_{\nu\sigma}}{\partial x_\mu} + \frac{\partial f_{\sigma\mu}}{\partial x_\nu}\right)^{[33]}$$

$$= \quad \cdot \quad + \frac{\partial}{\partial x_\mu}(f^{\ \mu}_{\mu\sigma}f^{\ \mu}_{\nu\sigma}) - f_{\nu\sigma}\frac{\partial f_{\mu\sigma}}{\partial x_\mu}$$

$$+ \frac{\partial^2}{\partial x_\nu}(f_{\mu\sigma}f_{\sigma\mu}) - f_{\sigma\mu}\partial f_\mu - \frac{1}{2}\frac{\partial}{\partial x_\nu}(f^{\ 2}_{\mu\sigma})^{[34]}$$

$$-k_\nu = \frac{1}{2}\frac{\partial}{\partial x_\mu}\underbrace{\left(\frac{1}{2}f_{\sigma\mu}f_{\sigma\nu} - \frac{1}{4}f^{\ 2}_{\sigma\tau}\ \delta^{\ \mu}_{\mu\nu}\right)}_{T_{\nu\mu}} \qquad \begin{vmatrix} \mu = 1 \quad \nu = 4 \\ f_{21}f_{24} + f_{31}f_{34} \\ \mathfrak{h}_z\mathfrak{e}_y - \mathfrak{h}_y\mathfrak{e}_z \end{vmatrix}$$

Man kann setzen

$$\left.\begin{array}{l} k_1 = -\frac{\partial T_{11}}{\partial x_1} - \frac{\partial T_{12}}{\partial x_2} - \frac{\partial T_{13}}{\partial x_3} - \frac{\partial T_{14}}{\partial x_4} \\ \text{— — — — — — — — —} \\ \text{— — — — — — — — —} \\ k_4 = -\frac{\partial T_{41}}{\partial x_1} - \frac{\partial T_{42}}{\partial x_2} - \cdot \quad - \frac{\partial T_{44}}{\partial x_4} \end{array}\right\} \quad \cdot \; k_\mu = -\frac{\partial T_{\mu\nu}}{\partial x_\nu}$$

Energietensor des elektromagnetischen Feldes.
Physikalische Bedeutung

$$\begin{array}{l} \not{k}_x = -\frac{\partial p_{11}}{\partial x} - \frac{\partial p_{12}}{\partial y} - \frac{\partial p_{13}}{\partial z} - \frac{\partial \not{s}_1}{\partial t} \\ \text{— — — — — — —} \\ \text{— — — — — — —} \\ \frac{i}{c}\Big| l = -\frac{\partial s_x}{\partial x} - \frac{\partial s_y}{\partial y} - \frac{\partial s_z}{\partial z} - \frac{\partial \eta}{\partial t} \end{array}$$

$T_{rs} = p_{rs}$ [u] $\not{s}_r = T_{r4}$
$s_{[s]} = \frac{i}{c} T_{[4s]} \quad T_{44} = -\eta$

Allgemeiner Satz: $k_\mu = -\frac{\partial T_{\mu\nu}}{\partial x_\nu}$

Für ⟨ abgeschlossen⟩ vollst. Syst. $\sum \frac{\partial T_{\mu\nu}}{\partial x_\nu} = 0$

Differentialform der Erhaltungssätze von Impuls-Energie
Übergang zu Integralform.

[p. 10] $T_{\mu\nu} = p\delta_{\mu\nu} + \rho_0 \langle\, dx_\mu \rangle u_\mu u_\nu$ [35] $u_\nu = \frac{dx_\nu}{ds}$ $\Sigma u_\nu^2 = 1$

$$-T_{44} = -p + \rho$$

$$\frac{\partial}{\partial x_{\langle\ \mu\rangle\nu}}(p\rho_{\mu\nu} + \rho_0 u_\mu u_\nu)$$

$$\frac{\partial p}{\partial x_{\langle\ \nu\rangle\mu}} + \langle\ \rho_0\rangle u_\mu \frac{\partial \rho_0 u_\nu}{\partial x_\nu} + u_\nu \rho \frac{\partial \langle\ \rho\rangle u_\mu}{\partial x_\nu} = 0$$

$$\frac{d\rho u_\mu}{ds}$$

$$u_\mu \Big| \quad \rho\frac{\langle \partial\rangle d\langle \rho\rangle u_\mu}{ds} + \frac{\partial p}{\partial x_{\langle \nu\rangle\mu}} + \langle \rho\rangle u_\mu \frac{\partial \rho u_\nu}{\partial x_{\langle \mu\rangle\nu}} = 0$$

$$\boxed{\frac{dp_{\langle \nu\rangle}}{ds} - \frac{\partial \rho u_\nu}{\partial x_\nu} = 0} \quad \Big| \qquad \text{Kontinuitätsgl.}$$

$$\frac{\partial p}{\partial x_\mu} + \rho\frac{du_\mu}{ds} + u_\nu\frac{dp}{ds} = 0 \text{ [36]} \quad \Big| \qquad K_\mu$$

Ruheenergiedichte = $-p + \rho = f(p)$

aus Zustandsgleichung

$$V_0 \quad M_0 \qquad p_0 = 0$$

$$E = M_0\langle +\rangle - \int_0^p p\,dV$$

$$-p + \rho = \frac{M_0}{V} - \frac{\int p\,dV}{V} = f(p)$$

$$= \frac{1}{V}\left(\frac{M_0}{\langle V\rangle}\langle +\rangle - pV + \underbrace{\int \langle p\rangle V dp \langle V\rangle}_{\int V dp}\right)^{+p} \Big| \quad \langle =\rangle \frac{1}{V}\left(M_0 + \int V dp\right) \text{ [37]}$$

$$= \rho_0 + \frac{1}{V}\int V dp$$

Überg. von Bew. Gl. zum Punkt

$$K_m V = \rho V_0 \frac{V}{V_0}\frac{du_\mu}{dt\sqrt{1-q^2}} = M\frac{du_\mu}{dt} = M\frac{d}{dt}\left(\frac{q_\mu}{\sqrt{1-q^2}}\right)$$

AD. [3 009]. The document, preserved in a notebook, 16 cm × 21 cm, consists of ten unnumbered pages. Page numbers provided here appear in the margin in square brackets. The title “Relativitätstheorie, Wintersemester 1918–1919” appears on the front cover. The five pages at the front of the notebook are transcribed in this document. Beginning on [p. 6], the notebook also contains lecture notes for the summer semester 1919, that are presented as Doc. 19. The notes for the winter semester continue upside down from the back of the notebook, presented here as [pp. 7–10]. A page has been torn out of the notebook between [p. 8] and [p. 9].

[1]This document is dated on the assumption that Einstein prepared these notes during his courses in winter semester 1918–1919 at the University of Berlin and the University of Zurich. The semester in Berlin was scheduled to run from 30 September 1918 to 1 February 1919 (see *Berlin Verzeichnis*

1918, title page), but Einstein left Berlin in early January. His comment, “Ende des Kollegs am Heftende (in Zürich notiert)” (see note 28) suggests that at least part of the notes were written for the course in Zurich which he held from 20 January until the second half of February 1919 (see notes 20 and 28).

[2]The exposition of special relativity and covariant electrodynamics in these lecture notes follows rather closely the treatment in an unpublished manuscript on special relativity from 1912 to 1914 (see Vol. 4, Doc. 1). In the following, this manuscript will be referred to as “Vol. 4, Doc. 1.” These lecture notes also show many similarities to the course on special relativity given by Einstein at the University of Berlin in the winter semester of 1914–1915 (see Einstein’s notes for this course in Vol. 6, Doc. 7). The lectures are dated. The first two dates given were Fridays, followed by one date on a Sunday, and the remainder on Saturdays. The order of topics covered shows one notable change over that of the lecture course given in 1914–1915 (Vol. 6, Doc. 7). In the earlier course the first two lectures covered a philosophical defense of the relativity principle, followed by the experimental underpinning for the ideas of special relativity, in particular the experiment of Fizeau concerning the velocity of light in moving water. Unlike previous expositions, in the present course Einstein begins with Maxwell’s equations and the theoretical formalism of Lorentzian electrodynamics, and he dispenses with the need to convince his audience of the complete integration of special relativity into the fundamental theory of electromagnetism. Only after this theoretical introduction does he go on to discuss the Fizeau experiment in the second lecture.

[3]*Heaviside 1892*, p. 199. See also Vol. 4, Doc. 1, p. 9.

[4]The rather dense calculation of an expression for the magnetic polarization current below is probably based on *Lorentz 1904b*, secs. 15, 28, 31, and 48. Note that in Vol. 4, Doc. 1, p. 18, Einstein avoids this more rigorous approach in favor of a simpler one based on a direct analogy with the case of an electrically polarized dielectric. In the following, $\mathfrak{q}$ is the magnetic dipole moment. Further down the page, when writing down the Maxwell-Lorentz equations for stationary matter, Einstein changes $\mathfrak{q}$ to $\mathfrak{m}$, his usual notation for the magnetic polarization vector.

[5]This should read $\acute{\imath}_m = c \operatorname{rot} \mathfrak{q}$.

[6]Here Einstein considers a magnetic element consisting of a current loop with circulating current $\mathfrak{J}$, area $\mathfrak{f}$, and normal $\mathfrak{n}$. From this he derives the magnetic dipole moment of the loop, $\mathfrak{q} = \frac{\mathfrak{J}}{c}\mathfrak{f}\mathfrak{n}$. On the next line, following the notation of *Lorentz 1904a*, sec. 28, he multiplies by *N* the number of magnetic elements, or current loops, per unit volume to write the magnetic dipole moment density of the material in question (the current loops representing bound electrons). The brackets around the $\mathfrak{q}$ may represent the averaged dipole moment per unit volume, presuming that the current loops are all identical, and (*N*) the average number of elements per unit volume. On the right in the next line is the expression for $\mathfrak{q}$ projected onto another normal (see the accompanying figure) whose angle to the first normal is written $n\alpha$.

[7]Here and in the line below Einstein calculates the current through an element of area $d\mathfrak{s}$, averaging over the orientation of the loops, and argues that it is equal to the averaged magnetic dipole density calculated in the line above (see note 6). A factor of *c* is missing in the right-hand side of the equation.

[8]On this line Einstein invokes Stokes’s theorem to show that the total current through a given surface area in a material, calculated two lines above (see note 7), can be rewritten in terms of a line integral around a closed curve surrounding the surface, with line element *dS*. The factor of *c* on the right-hand side of the equation should be omitted. This gives directly, on the next line, an expression for the magnetic polarization current of a material in terms of $\mathfrak{q}$ (see note 6), which should read $\acute{\imath}_m = c \operatorname{rot} \mathfrak{q}$.

[9]*Rowland 1878*, *Rowland and Hutchinson 1889* (see Vol. 4, Doc. 1, note 8).

[10]*Röntgen 1888* and *Eichenwald 1903*, *1904* (see Vol. 4, Doc. 1, p. 17, note 21, and Vol. 6, Doc. 7, note 11). See, for example, *Laue 1913*, §2, and *Pauli 1921*, sec. 36, for discussions of these experiments as well as Wilson’s (see note 11).

[11]*Wilson, H. 1904* (Vol. 4, Doc. 1, p. 17, note 22, and Vol. 6, Doc. 7, note 8).

[12]*Fizeau 1851*. See Doc. 31, [pp. 1–2] and [p. 13], for a discussion of the importance of Fizeau’s experiment.

[13]This argument, which goes as far as the bottom of this page of the manuscript, follows the calculation on pp. 25–27 of Vol. 4, Doc. 1, of the Maxwell-Lorentz equations for a slowly moving particle. Note that $\mathfrak{q}$ now, and throughout the rest of the document, refers to the velocity of a given mass or object.

[14]The following derivation of the Fresnel coefficient follows the one given in *Lorentz 1892a*, pp. 524–526. See also Vol. 4, Doc. 1, pp. 27–28, and Vol. 6, Doc. 7, pp. 47–48.

[15]This insert should be "$-V/\ c$" if, as it appears, the term it is supposed to amend should read "$-\left(\frac{\mathfrak{q}}{c}-\frac{V}{c}\right)\mathfrak{d}'$."

[16]The middle term in the top row of the matrix should be "$\frac{\mathfrak{q}}{c}-\frac{V}{c}$."

[17]Einstein here writes down the right formula in this derivation, correcting the error in the matrix highlighted in note 16. Presumably he corrects the error from memory, knowing that the cross term $2\frac{V\mathfrak{q}}{c^2}(\varepsilon-1)$ does not cancel out as the matrix he has written down would have it. The deleted equation immediately below the matrix also silently corrects the error. In this equation $\mathfrak{q}$, the speed of the running water (or other moving dielectric) in Fizeau's experiment is treated as a quantity much smaller than *c*.

[18]Here V_0 is the speed of light in water at rest and Δ is the correction to the velocity of light in a frame in which the water is moving, where Δ is much smaller than V_0.

[19]On 9 November 1918, mass demonstrations by armed workers in Berlin precipitated the resignation of the imperial government. Soon after Einstein, Max Born (1882–1970), Extraordinary Professor of Physics, and Max Wertheimer (1880–1943), Professor of Psychology, both at the University of Berlin, made their journey through revolutionary Berlin to the Reichstag to appeal for the release of some professors from the University of Berlin who had been incarcerated by revolutionary students of the university (see Einstein to Pauline Einstein, 11 November 1918 [Vol. 8, Doc. 651], note 3 and references therein, as well as *Born 1975*, pp. 257-259).

[20]This is the last date given in the notes and must have been the last lecture that Einstein intended to deliver, since by 9 January 1919 he was already in Zurich, having left Berlin earlier than anticipated "for pressing reasons" ("aus dringlichen Gründen"; Proclamation of Erziehungs-Direktion des Kantons Zürich, 9 January 1920, SzZU, Rektorats-Archiv, No. 208-10). Einstein went to Zurich to give a course of lectures at the University of Zurich on the same topic as these lectures (see Einstein to Heinrich Mousson, 17 December 1918 [Vol. 8, Doc. 674]).

The remaining material in the notes covers the material of Vol. 4, Doc. 1, sec. 4, on the electrodynamics of moving bodies, following it quite closely.

[21]This equation should be "$\frac{\partial(f_{\mu\nu}+p_{\mu\nu})}{\partial x_\nu} = \mathfrak{J}_\mu$."

[22]The third term "$\frac{\partial f_{\rho\mu}}{\partial x_\mu}$" should be "$\frac{\partial f_{\rho\mu}}{\partial x_\nu}$."

[23]Under the empty square root signs on this line and the one below it, there should be "$1-q^2$."

[24]The two terms indicated by dashes here would read "$\mathfrak{h}_z\frac{\mathfrak{q}_y}{\sqrt{1-q^2}}-\mathfrak{h}_y\frac{\mathfrak{q}_z}{\sqrt{1-q^2}}$."

[25]"$\mathfrak{h}_1$" should be "$\mathfrak{h}_x$." In this term and the one immediately above, the missing term is $1-q^2$. On both of these lines, Einstein has neglected to replace components of $\mathfrak{J}_\mu$ with the components of the four-velocity u_μ.

[26]Beginning with $\mathfrak{m} = (\mu-1)\mathfrak{h}$ (Vol. 4, Doc. 1, p. 12, eq. (8)) and taking $\mathfrak{h} = \mathfrak{b}-\mathfrak{m}$, one obtains $\mathfrak{m} = \frac{\mu-1}{\mu}\mathfrak{b}$ rather than the result Einstein derives here.

[27]In full, this equation would read $p_{\mu\nu}u_\rho+p_{\upsilon\rho}u_\mu+p_{\rho\mu}u_\nu = -\frac{\mu-1}{\mu}(f_{\mu\nu}u_\rho+f_{\upsilon\rho}u_\mu+f_{\rho\mu}u_\nu)$,

if one makes the correction in note 26. This version of the equation, together with Einstein's boxed equation immediately above, agrees with, e.g., eqs. (286) of *Pauli 1921*, p. 662.

[28]Einstein completed his notes for this course after arriving in Zurich, presumably as an aid for the lecture course on the same topic he gave there (see note 20). The lecture notes on general relativity, which start on the same page of the notebook as the one on which this phrase is written, are presented as Doc. 19.

[29]These components of the electromagnetic field tensor $f_{\mu\nu}$ should be "$f_{23} = \mathfrak{h}_x$," "$f_{31} = \mathfrak{h}_y$," and "$f_{12} = \mathfrak{h}_z$."

[30]Here "$\mathfrak{J}_\nu$" should be "$\mathfrak{J}_\mu$."

[31]The first and the third $\mathfrak{i}$ should be the imaginary unit *i*. The quantity *l*, which differs from the notation used either in Vol. 4, Doc. 1, or in Vol. 6, Doc. 7, is the flux of electromagnetic energy density which is absorbed by a receiver.

[32]"$\mathfrak{k}$" should be "$\mathfrak{k}_x$."

[33]The term on this line is equal to the second term in the equation above, $f_{\mu\sigma}\frac{\partial f_{\mu\nu}}{\partial x_\sigma}$, by Maxwell's second system of equations. Einstein is deriving the form of the electromagnetic energy-momentum tensor $T_{\nu\mu}$.

[34]The undeleted term on this line, together with the two terms from the line immediately above, sum up to $f_{\mu\sigma}\frac{\partial f_{\mu\nu}}{\partial x_\sigma}$ (see preceding note). To continue the derivation one moves the term $-f_{\nu\sigma}\frac{\partial f_{\mu\sigma}}{\partial x_\mu}$ to the left-hand side. By changing the names of the summation variables μ and σ (hence the superscript μ indices on the line above, positioned above σ indices), one consolidates terms on both sides and can divide by two to recover the expression for k_ν in the line below. See *Einstein 1916b* (Vol. 6, Doc. 27), sec. 2.

[35]For Einstein's explanation of this formulation of the energy-momentum tensor for a perfect fluid, see *Einstein 1916e* (Vol. 6, Doc. 30), p. 811.

[36]The third term should be "$u_\mu\frac{dp}{ds}$."

[37]Einstein deleted the equals sign on this line. Nevertheless, the expressions to the left and right of the vertical line are equal, if one includes the "$+p$" term written above the line. The expression on the left of the vertical line was derived from the line above using integration by parts.