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“A Simple Application of the Newtonian Law of Gravitation to Globular Star Clusters”

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56. "A Simple Application of the Newtonian Law of Gravitation to Globular Star Clusters"

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There is hardly any doubt that Newton's law may be extrapolated beyond the distances for which it has been verified. This confidence is also supported by the general theory of relativity, which provides a rational foundation for Newton's law, so that an extrapolation to larger distances of bodies acting on each other seems even more justified. However, the general theory of relativity predicts considerable deviations from the Newtonian laws in the case of a spatially finite universe — yet only in the case where the mean density of the stellar matter in the gravitating structure under consideration is not substantially larger than the mean density of the universe in general.

In what follows, the Newtonian law of gravitation is applied to so called globular star clusters. The main difficulty here is to arrive at some testable consequences, since our actual knowledge about the motion of stars in such a cluster is exceedingly limited. The positional changes of the stars during the time spans available to us are too small to be noticeable with our present means of observation. Furthermore, over these long distances, the stars are insufficiently bright to allow for an investigation of their motions in the direction of the line of sight by means of Doppler's principle. All one has is the image of the star cluster, projected parallel to the line of sight, and even this only for the brightest stars in the cluster.

Nevertheless, one knows the approximate distance of such globular clusters from us, and consequently their approximate true radius. This estimate is based on the proven assumption that stars of the same spectral type are approximately equal in size and approximately equal in absolute brightness. This assumption allows us to derive, from the apparent luminosity of stars, a conclusion about their distance from us, namely, by comparing them to stars of the same spectral type in our vicinity. If one now knows the distance of these neighboring stars, then one can also obtain the distance of the star cluster from us. From the apparent radius of the star cluster follows—at least for the order of magnitude—the true radius of the star cluster; in this manner, values of 100 - 500 light years resulted for the latter.

It is probably safe to assume, that the bright stars of the star cluster approximately correspond to the absolutely bright stars in our neighborhood. For the latter it has been found—using Doppler's principle—that they move relative to each other with a mean velocity of about 26 km/sec[1], and we can probably assume that this is also the order of magnitude of the mean velocity of the bright stars of the stellar cluster relative to its center of gravity, the more so as it has been shown that the mean velocities of stars of different spectral type also agree in their *order of magnitude* with each other.

We now assume that the distribution of stars in a stellar cluster is stationary, insofar as the latter does not substantially change its radius and its star distribution (from a statistical point of view) during a time period in which the individual stars of the cluster traverse a (curvilinear) path that is large compared to the radius of the cluster. There is hardly any doubt, that this condition is satisfied for the radially symmetrical, and for many star clusters, similar statistical distribution. Then it is possible to apply Clausius's virial theorem to the star cluster as a whole, by treating individual stars as material points. In the case of Newtonian forces this yields, as H. Poincaré probably was the first to show,

$$L = \frac{1}{2}\Phi \ . \tag{1}$$

L stands for the combined kinetic energy of all the stars in the cluster; Φ is the negative potential energy attributed to the cluster if the zero point of the potential energy of the stars is defined such that it vanishes when the distance between stars approaches infinity.

In order to be able to draw conclusions from equation (1), I make approximate assumptions about the structure of the cluster. I treat those stars of the cluster that are pictured on the photographic plate under short exposure as being of equal mass m, and N shall be the total number of this type of stars in the whole cluster. Furthermore, I assume, for the time being, that the less luminous stars, that is, also the smaller ones in the cluster, do not contribute substantially to the gravitational field

[1] More precisely: relative to the center of gravity of the system to which they belong.

of the cluster, such that they can be neglected in the calculation of L and Φ. The immediate result then is, if v is the (quadratic) mean of velocity $\left(v = \sqrt{\overline{v^2}}\right)$,

$$L = N \cdot \frac{mv^2}{2} . \qquad (2)$$

For the calculation of Φ one must know the spatial density ρ for the stars of the cluster. It is well known that it can be represented in a satisfying manner by the empirical formula:

$$\rho = \frac{3}{4\pi} \frac{N}{a^2} \left(1 + \frac{r^2}{a^2}\right)^{-\frac{5}{2}} . \qquad (3)$$

Here, a is a length proportional to the radius of the cluster, 2 a is the radius where the density has dropped to about 2% of the central density. Furthermore, ρm is the mean density of the stellar matter within the cluster at a given point. One does not commit a substantial error if one calculates Φ as if matter were distributed continuously with the density ρm. This results in:

$$\Phi = A \frac{kN^2 m^2}{a} . \qquad (4)$$

k is the gravitational constant, A is a numerical factor which I find to be approx. 0.6.

For the radius of the star cluster one gets from (1), taking (2) and (4) into account,

$$2a = 1.2 \frac{kNm}{v^2} . \qquad (5)$$

If one sets for the star cluster in Hercules $N = 2000$, $m = 15$ masses of the sun, $v =$ 26km/sec, one obtains

$$2a = 0.65 \cdot 10^{18} cm = 0.65 Lightyears.$$

According to the apparent brightness of the brightest stars of the cluster, one must assume the distance from us such that its radius cannot be less than 100 light years. Therefore, there must be an error in our assumption.

I had the opportunity to discuss the present difficulty in detail with my colleagues at the Astrophysical Institute in Potsdam. The result was that, according to our present knowledge of the masses and distribution of fixed stars *one* of my assumptions is significantly wrong. The vast majority of the fixed stars in a star cluster may have a considerably lower luminosity than the approximately 2,000 stars which are pictured on the photographic plate under short exposure, without, however, having to assume that their mass is substantially smaller than that of the brightest stars. From pictures of the star cluster with long exposure time, and also from the distribution of fixed stars in our neighborhood, one can estimate that the number of fixed stars that contribute to the gravitational field of the cluster is about 100 times larger than we have assumed above. We then obtain a radius of the star cluster of 65 light years, a value that is not so far off the lower limit that has been estimated using a different method.

The incompleteness of the currently available observational material forces us for the time being to be content with this agreement of the order of magnitude. More precise results are dependent on a better knowledge of star masses and star velocities. But *one* conclusion of considerable interest can be drawn from the agreement of the order of magnitude, namely, that the non-luminous masses contribute no higher order of magnitude to the total mass than the luminous masses.